# VASTNESS REVISITED

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**ABSTRACT**

In 1985 Terrence Langendoen and Paul Postal argued that the output of natural grammars, that is natural languages, had a degree of infinity that was larger than that of any set in mathematics. While their work seemed arcane, and has not been widely taken up since, they did address a fundamental question that lies at the formal heart of generative grammar: what size of set does a grammar generate, or, how "big" is language?

McCawley (1987) criticized their work on the grounds that an infinitely long sentence was ungrammatical, his objection being a linguistic version of what in logic is termed the halting problem for a Turing machine. Langendoen and Postal could bypass McCawley's criticism by taking refuge in the distinction between a potential and an actual infinite. The nature of generative grammar would remain the same in either case.

In fact there is a hitherto unrecognized flaw in Langendoen and Postal's work that arises when they filter the output of their "machine" (a power set operation) that makes bigger sets from smaller ones (pp. 56-7). The resulting filtered set reverts back to a size, or "cardinality," that is the same as the original set of infinitely long sentences, this cardinality being what in math is called "denumerable," or in set theory "aleph-null." Their effort to advance language to unbounded infinite heights stalls at the lowest level of infinity, aleph-null. Their hierarchy cannot be extended.

A simple and natural model of language use is put forward here: language is used in context to produce acceptable or unacceptable instances of a utterance – context pair. The context model is an extensional one that uses space-time as a natural base for an abstract space of "speech." It can then be shown that the human ability to use language uses a generative capacity whose output is greater than aleph-null, that is, its cardinality is at least as large as that of the continuum (aleph-1) and probably that of all relationships on the continuum (aleph-2).

These findings form a natural generalization and extension of the Chomsky hierarchy to "super-grammars." They also complete Chomsky's initial demands for generative grammar: an explanation not only for the infinite use of language, but also its spontaneous, and creative use, the last two of which have to date only been partially met.

Keywords: Vastness, recursion, generative paradigm, Chomsky hierarchy, performance, set theory, Turing test, Turing machines, degrees of infinity, super grammars.

**1. INTRODUCTION.** The formal structure of Generative Grammar may be represented simply as (1):

(1) $G \rightarrow \{S\}$ XOR $\{*S\}$,

where G is the grammar that orders some set of data as either acceptable sentences, {S}, or unacceptable, {*S}, by assigning derivations to members of the acceptable set. Here XOR is the logical symbol or abbreviation for 'exclusive or.'

1.1. Some linguists may desire a system with degrees of grammaticality, thus complicating the picture in (1), but this demand could be accommodated as in (2a) where G assigns utterances to a graded series of sets, $\{S_n\}$, with a threshold of acceptability reached at some value of the index, let us say, k. This threshold can be used to partition the outcome again into two sets (of sets) (2b), duplicating the condition in (1).

(2) a. $G \rightarrow \{S_n\}$

b. $G \rightarrow \{S\}$ XOR $\{*S\}$, where $\{S\} \supset \{S_i\}$, $1 \leq i \leq k$, up to $S_k$ acceptable, and $\{*S\} \supset \{S_j\}$, $k+1 \leq j \leq n$ unacceptable.

1.2. Crucially G is finite, while both {S} and {*S} are infinite. This is Chomsky's criterion for generative grammar, making infinite use of finite means (Chomsky 2006; 1975). Further, Chomsky sought to capture through this generative paradigm the infinite, spontaneous, and creative use of language. Iteration is assumed to explain the infinite nature of language,(the "size" of {S} in a sense that I shall make exact). I shall conclude by arguing otherwise, though I admit iteration and the denumerable nature of sentence structure. Iteration is not why language "feels" infinite. Nor does iteration nor the formal schema in (1) in any way explain the spontaneous or creative use of language. One must seek the infinite feel of language in its use in context, that is in the performance of language, and it is there that spontaneity and creativity emerge.

**2. CARDINALITY**. Langendoen and Postal (1984) took up this fundamental feature of generative grammar as depicted in (1) and by doing created a new field of linguistics, "vastness" theory. They generated a set of sentences, $\boldsymbol{S_o}$, with cardinality $\aleph_0$, ("aleph-null," or "aleph zero"), the cardinality or "size" of the integers, these constituting a denumerably infinite set. They achieved this through the iteration of embeddings. To capture their effort symbolically, as in (3), let **E,** an operator, act on a finite sentence, σ, of length k, such that with $\mathbf{E}^n(\sigma)$ in the limit as $n \rightarrow \infty$. Then their findings can be represented as in (3).

(3) a. $\mathbf{E}^n(\sigma) = \mathbf{E}(\mathbf{E}(\mathbf{E}\ldots(\sigma))\ldots) = \Sigma$ "n times,"

b. $\mathbf{card}(\Sigma) = \mathbf{card}(\mathbf{E}^n(\sigma)) = \aleph_0 \otimes k = \aleph_0$ ,

where **card** is a "cardinality operator," a "machine" that counts the size of a set, and $\aleph_0$ is a "denumerable" infinity, a set that can be counted with the integers, **Z**. The ⊗ symbol stands for the cartesian product of two sets, here represented by their two cardinalities.

2.1. One should note that $\aleph_0$ is a "transfinite cardinal," in fact, it is the least such cardinal. Transfinite cardinals are a generalization of the concept of size (Cantor 1955). When multiplying with a transfinite cardinal, as in (4a), (or with a series of transfinite cardinals), the biggest one always prevails (Devlin 1979: 82-8; Enderton 1977: 138-44; Kamke 1950: 17-51).

2.2. The actual sentence which they used was that in (4), simple but adequate for their formal purposes.

(4) $\boldsymbol{S_o}$ = "I know that I know that … Babar is happy."

The infinity is in the "middle" of the sentence, so to speak. Their sentence has a beginning and an end, but an infinite middle, much like an infinite series approaching a limit as well as being defined by or equated to that limit. This choice enabled Langendoen and Postal to avoid the criticism that they had created a sentence without an end, and therefore one that was non-grammatical, though in fact such a criticism was leveled against them. The sentence in (4) consists of discrete words assembled into discrete clauses, and as such each word can be assigned an integer, starting with 1 at the beginning. In this way the sizw of the sentence, strictly speaking its cardinality, is in one to one correspondence with the integers. Hence the sentence has cardinality $\aleph_0$, which by definition is that of the integers, **Z.**

**3. Remarks on cardinality**. To avoid controversy $\boldsymbol{S_o}$ must be viewed as a feature of competence, subject to a performance "truncation" (which can be denoted by ◊). In other words, one can never utter an infinite sentence, but it is in the nature of G that such a sentence could be assigned a derivation, that is, that it could be assigned to either {S} or {*S}. One simply must stop because of performance constraints, that is, one must truncate one's sentence, ◊.

3.1. This problem of truncation is not unique to language. The same must be the case with the infinitude of the transcendental numbers. For example, π (the ratio of a diameter of a circle to its circumference), or *e* (the base of the natural logarithms) can never be represented, but must be understood. The symbols, 'π,' '*e*,' etc., are in fact limits. Truncation looks simple enough, but it formalizes a solution to the long standing problem of real as opposed to potential infinities.

3.2. James McCawley noted (1987) that infinitely long sentences could be argued to be inherently ungrammatical. McCawley may have had in mind the halting problem for a Turing machine, usually symbolized as $M_T$. In modern experience this would be equivalent to an application freezing up on a computer, even if a well-defined end or goal exists. Langendoen and Postal deal with sentences that are "locally grammatical," that is, any given phrase taken within a finite "sampling window" will be grammatical. One can then form an infinite union of all such locally grammatical substrings to produce a "globally grammatical" string, (5), even if it is "unacceptable" in the "McCawleyan-Turing" sense, that is, G or $M_T$ fail to halt.

(5) a. $G \rightarrow \sigma, \sigma \subset \{S\}$ (local grammaticality)

b. $\cup (\sigma_\iota) = \boldsymbol{S_o} \mid \boldsymbol{S_o} \subset \{S\}, 1 \leq i \leq \aleph_0$ (global grammaticality)

With truncation, however, we can admit Chomsky's recursion into the structure of grammatical competence without running afoul of either performance constraints nor Turing's halting criterion.

**4. ASIDE ON SET THEORY**.[1] For what I wish to prove the concept of a power set, (6), is crucial. This is the set of all sets that can be formed from the elements of a given set. Among these sets are the null set, ø, and the whole set itself. It is as though one were to take a bad of fruit and choose all possible combinations of the fruit therein, including no choice, the null set, or the entire bag itself. Every choice leaves behind a remainder, which itself is a possible choice, so the choices come in pairs. This pairing of choices explains the general result in (8).

(6) i. $A = \{a, b, c\}$

ii. $\wp(A) = \{ø, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{c, b\}, \{a, b, c\}\}$

where $\wp$, the power sets operator, makes bigger sets from smaller ones.

The size of A, its cardinality, is merely the number of elements contained within it, as in (7). Similarly the cardinality of A's power set can be determined by simple counting, though now we can relate the two sizes, as in (8).

(7) $\textbf{card}(A) = 3$

(8) $\textbf{card}(\wp(A)) = 8 = 2^{\textbf{card}(A)}$

The relationship in (8) is general.

4.1. So, (8) may be extended to relationships such as (10), where we have assumed that the power set operator has once again faithfully created a set that is larger than its input.

(9) $\textbf{card}(\text{integers}), \textbf{card}(Z) = \aleph_0$

(10) $\textbf{card}(\wp(\aleph_0)) = \aleph_1 = 2^{\aleph_0}$

This is Georg Cantor's "Continuum Hypothesis," usually abbreviated as CH, which is controversial (Cohen 1966). So, if one wishes, one may simply have:

(11) $\textbf{card}(\wp(\alpha)) = \beta_, \beta > \alpha$.

[1] See Enderton, Kamke, Devlin, Kunen, Kanamori 2000

In what follows I shall assume the CH. (The CH may be generalized. See (14) below.)

4.2. One may add a decimal ( a set {.} of cardinality 1) to the counting numbers, **Z**, (ø, 1, 2, …). As with transfinite multiplication the transfinite number prevails in addition (here denoted by ⊕), so that $\aleph_0 \oplus 1 = \aleph_0$. One can then shuffle this new set of **Z** ∪ {.} (a "shuffle product," see Eilenberg 1974, p. 19). The result looks like the real numbers, **R**, the numbers on the continuous line, such as 2.718281828…, (*e*, the base of the natural logarithms), 3.141592653…, (π), or 4.472135955…,(the square root of 20). Cantor denoted the cardinality of the "reals" or of the continuum by $\aleph_1$.

4.3. Many mathematicians find this seemingly sensible result controversial. Paul Cohen (1966) proved that the CH is independent of the other axioms of set theory, even though his famous forcing argument seems to assume the consequent. In other words, one can have set theory with or without the CH.

**5. CLIMBING THE TRANS-FINITE LADDER**.[2] Langendoen and Postal ambitiously argue that language climbs the "transfinite" ladder (Enderton, p. 9, fig. 3). To do so they form a power set of $\boldsymbol{S_o}$, $\wp(\boldsymbol{S_o})$. They then construct a new set, $\boldsymbol{S}_1$, putatively equinumerous (of the same cardinality) with $\wp(\boldsymbol{S_o})$. This new set consists of sentences from $\wp(\boldsymbol{S_o})$ rendered grammatical by means of conjunction, since the elements of $\wp(\boldsymbol{S_o})$, being scrambled from $\boldsymbol{S_o}$, are ungrammatical:

(12) $\wp(\boldsymbol{S_o})$ = {[I know [that B…]], *[[that B is happy] I know [that B is happy]], …}

(13) $\boldsymbol{S}_1$ = {[I know [that B…]], [[I know [that B…] and [I know [that I know [that B…]]]], …]

5.1. They assume the CH. (Once the CH is assumed, the generalized CH, or GCH, may be employed and the set theory will remain consistent.) They then assert that $\boldsymbol{S}_1$, as with $\wp(\boldsymbol{S_o})$, has the power of the continuum, $\aleph_1$. This is only their first step with the power set operator. They then form the power set of $\boldsymbol{S}_1$, and proceed ad infinitum. In this way they claim to ascend the transfinite ladder, by means of the "generalized continuum hypothesis," (14).

(14) $\aleph_{n+1} = 2^{\aleph_n}$

They conclude that language has truly extraordinary cardinality, beyond the size of any set.

[2] See Enderton: 7-9

**6. STUCK ON NULL, $\aleph_0$.** $\wp(\boldsymbol{S_o})$ is not well ordered lexicographically (Kunen 1980: 173-5). To proceed up the transfinite hierarchy and retain grammaticality, Langendoen and Postal's must clean up the output of their power set operation. $\boldsymbol{S}_1$ is supposed to salvage grammaticality from the chaos of $\wp(\boldsymbol{S_o})$, but their way of forming $\boldsymbol{S}_1$ from $\wp(\boldsymbol{S_o})$ can be shown to limit the cardinality of $\boldsymbol{S}_1$ to $\aleph_0$. Any further iteration in the manner they envisage fails to create sets of higher cardinalities because of the necessity of each time filtering out ungrammatical forms generated through $\wp(\boldsymbol{S_n})$. In effect their formation of $\boldsymbol{S}_1$ can be shown to be achieved through a denumerable application of conjunction, $\wedge$, which may be viewed as an operator acting $\aleph_0$ times simply on $\boldsymbol{S_o}$, so that each element of $\boldsymbol{S}_1$ in (13) consists of a conjunction of finite sentences. The whole is therefore composed of an infinite conjunction of these elements. If one allows the conjoined sentences themselves to proceed toward an infinite limit, then at most one has:

(15) $\textbf{card}(\wedge(\boldsymbol{S_o})) = \aleph_0 \otimes \aleph_0 = \aleph_0 = \textbf{card}(\boldsymbol{S_o})$.

No further ascent into the transfinite beyond is possible, since any power set must be filtered for grammaticality through the conjunction operator.

**7. SUPER GRAMMARS (TURING TEST GRAMMARS).** While the cardinality of the sets in (1) remains denumerable, this technicality does not directly shape our sense of language's infinitude. I take a different approach, one based on performance, and will show that the sense of language's infinity is instead captured by appropriate use," that is by taking language in context, with no "final" use being self-evident.

7.1. I posit a pair consisting of two elements:

(16) (C, U) , with C = context, and U = utterance.

'Appropriate use' means to parse the set {(C, U)} into acceptable {(C, U)} and unacceptable {*(C, U)} sub-sets by the generative effects of Γ, a "super-grammar." Γ is equivalent to the Turing test for artificial intelligence, wherein one may speak to a machine without being able to distinguish it from a human. Hence I am tempted to use an alternate designation, "Turing Test Grammar." The process in (17) is superficially analogous to that in (1).

(17) $\Gamma \rightarrow \{(C, U)\}$ XOR $\{*(C, U)\}$

A determination of the cardinality of the sets in (17) will show that Γ is fundamentally different from G.

**8. THE CARDINALITY OF {(C, U)}.** Generative capacity is a measure that depends both on the Kolmogorov complexity (Li and Vitányi) of a grammar's output (complexity of the derivations or structures assigned) and upon the cardinality of that output. For shorthand I shall simply assign an "extended" cardinality to a grammar, (18) and (19), equal to that of its output(s), though strictly speaking it should represent the number of processes or rules and the size of the lexicon contained in a given G and accordingly be a finite, in fact, relatively small number. In (18) I take G to be $G_0$, the most complex grammar of the Chomsky hierarchy, though each grammar of the hierarchy has the same cardinality.

(18) $\mathbf{card}(G_0) \equiv \mathbf{card}(L_0) = \mathbf{card}(\{S\}) = \aleph_0$

For the super grammar in (19) its extended cardinality is that of its "super-language," Λ, which is the product of the components of Λ, namely the context and the utterance.

(19) $\mathbf{card}(\Gamma) = \mathbf{card}(\Lambda) = \mathbf{card}(\{(C, U)\}) = \mathbf{card}(\{C\}) \otimes \mathbf{card}(\{U\})$

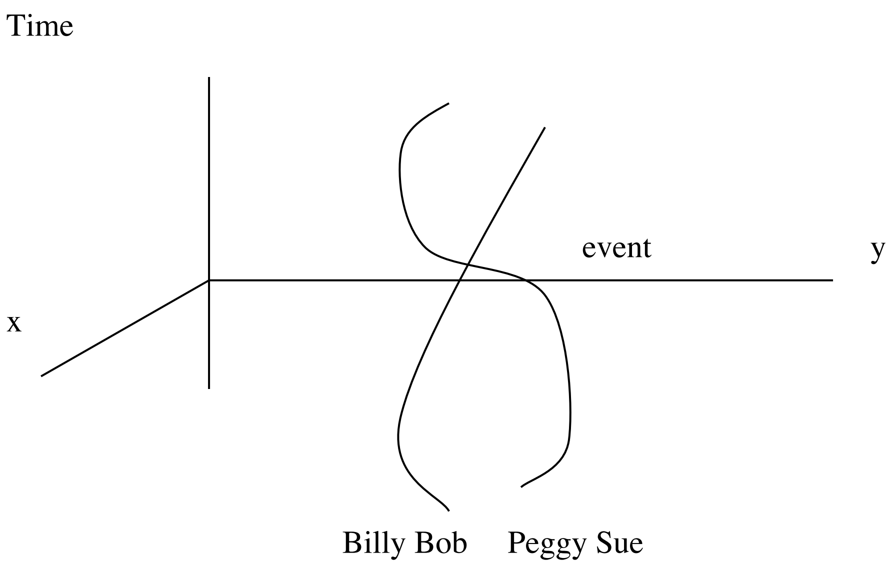
Whatever grammatical paradigm is used to model is {U}, its cardinality is plainly that of $L_0$, as in (20).

(20) $\mathbf{card}(\{U\}) = \aleph_0$

Clearly the answer to (19) depends upon a reasonable model of context.

8.1. I will take {C} to be set of all contexts in life, which has the world as its stage. More specifically contexts take place in space-time, Σ. In (21) I depict the world lines, as four dimensional representations are called, of two people, let us call them Billy Bob and Peggy Sue, as they move through space and progress through time.

(21) World-lines in Space-time, Σ, (two people)



Such world lines, however, depict the mere physical aspects of Billy Bob and Peggy Sue. While contexts are certainly set in Σ, they also are constructed from cultural perceptions and conceptions. Significant events or states are functions of these contexts. I shall term these significant events or states "histories," $\mathscr{H}$. One might view a history, **H**, as a function of individuals, "egos," in a context, **C**, as in (23).

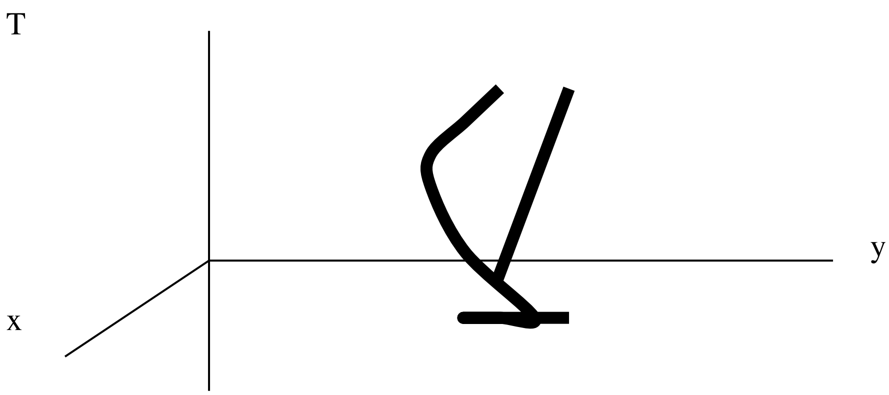
(23) $\mathbf{H} = f(\{\text{ego}\}, \mathbf{C})$

The cardinality of {ego} is finite, since there are only so many people in history.

8.2. World-lines are continuous, but the elements of $\mathscr{H}$ can be discontinuous. In fact histories begin, end, intersect, and avoid one another, all of which conditions can be highly significant for the parties concerned. In (24) I have drawn (crudely) the legal context of Billy Bob and Peggy Sue's marriage. The bottom bar represents the moment when Billy Bob and Peggy Sue are married in the company of family, followed by a brief honeymoon, (the unified stretch), and then by more normal, independent married life, all of which has legal significance. Their prior, independent lives are not relevant to legal considerations and therefore are not a part of the history depicted in (24).

(24) History in Space-time

(the two are married)




8.3 The histories, $\mathscr{H}$, are in effect subsets of Σ, and combinations of subsets of Σ. To achieve $\mathscr{H}$ one must take the power set of Σ:

(25) $\mathscr{H} = \wp(\Sigma)$

It is important when forming a power set of a continuum that it be done so as to permit non-continuous or non-connected forms, otherwise a mere diffeomorphism (distortion) results and the cardinality is not increased. (25) achieves this because of the conditions on histories. Technically, space-time Σ serves as the "base space" for the "fiber bundle" of context $\mathscr{H}$, (Morita: 232; Frankel: ch. 17). We may now calculate the cardinality of {C}.

8.4. Assuming the CH, the cardinality of space-time Σ is:

(26) $\mathbf{card}(\Sigma) = \aleph_1$ .

That of all possible histories, these being functions of individuals in contexts set within Σ is:

(27) $\mathbf{card}(\mathscr{H}) = \mathbf{card}(\wp(\Sigma)) = 2^{\aleph 1} = \aleph_2$ .

The result in (27) implies that the cardinality of Λ, that is, of {(C, U)} is:

(28) a. $\mathbf{card}(\mathscr{H}) = \mathbf{card}(\{f(\{\text{ego}\}, \mathbf{C})\}) = \mathbf{card}(\{\text{ego}\}) \otimes \mathbf{card}(\{C\}) = \mathbf{card}(\{C\}) = \aleph_2$

**b.** $\mathbf{card}(\Lambda) = \mathbf{card}(\{(C, U)\}) = \mathbf{card}(\{C\}) \otimes \mathbf{card}(\{U\}) = \aleph_2 \otimes \aleph_0 = \aleph_2$

The result in (28) implies that our ability to use language spontaneously and creatively is beyond the power of any deterministic algorithm or a $G_0$ ($M_T$), since these produce only denumerable sets, $\aleph_0$.

8.5. Therefore the extended cardinality of Γ is $\aleph_2$. This is equivalent to the set of all possible functions and relations on the continuum. Such a system would seem spontaneous and creative precisely because no deterministic grammar can achieve this level, nor can one achieve $\aleph_1$ . An algorithm with a random application might achieve the power of the continuum in capacity if not execution, $\aleph_1$. One might now reasonably ask what conceivable sort of machinery lies within Γ.

**9. Representations of a Super Grammar**. A super grammar matches contexts with utterances, so one might treat these as an inner product of infinitely dimensional vectors, that is as a Hilbert Space (Young):

(29) $\Gamma = \{\langle C|U \rangle\}$

One could then assign an "acceptability measure" to these inner products, with a threshold of acceptability, much as in (2):

(30) $\Gamma \rightarrow \boldsymbol{\mu}(\langle C|U \rangle)$, if $\boldsymbol{\mu} <$ threshold $\tau$,

then $(C, U) \in \{*(C, U)\}$

In other words if the usefulness of the utterance in a given context falls below a threshold of approrpiateness, $\tau$, with $\tau$ being determined empirically, socially, or logically, then the utterance is not acceptable or the context has been misconstrued by the speaker.

9.1. Symmetries within {U}, that is, grammatical constraints, and symmetries within {C}, perceptual and cultural constraints, would then dictate the outcome of $\Gamma$ under the acceptability measure. Such constraints have been the subject of generative grammar for several decades now, while similar perceptual and cultural constraints have been the object of study by cognitive scientists and some anthropologists. The effective study of super grammars would require a new degree of collaboration among these three parties.

**10. Speculations.** Work by Gödel on the hierarchy of infinite languages (Kanamori 2007) suggested that such super-grammars might, as a formal class, be subject to extension up the scale of transfinite numbers, as Langendoen and Postal once hoped for $G_0$. If we equate or rename $G_0$ as $\Gamma_0$, then we might extend and generalize the Chomsky hierarchy as in (31).

(31) $G_0 = \Gamma_0 < \Gamma_1 < \Gamma_2 < \ldots < \Gamma_n < \ldots$

$\Gamma_1$ might be our ability to cope with space-time, while $\Gamma_2$ would be our ability to make sense of our lives, to formulate histories in contexts. Any higher super grammar would seem to lie beyond the current level of the human mind. $\Gamma_3$ and above would be features of a genuine super mind.

10.1. The inner product in (29) bears a superficial resemblance to the Dirac notation used in quantum mechanics (for example, Liboff: 93-9). If taken seriously, this model suggests an "anti-Cartesian" paradigm, namely, that the universe is more like a mind than it is like a clockwork. Randomness is an inherent feature in both. Without randomness, or "frozen accidents" (Hartle: 46, quoting Murray Gell-Mann) in mind and the physical world, existence would have no informational content. All would be predictable and therefore lack information, as the definition of information in (32), when applied to certainty, yields zero. Here P is the probability of an event occurring. Since this ranges from zero (cannot happen) to one (certainty), P is normally negative. The minus sign therefore makes the information, I, a positive number.

(32) $I = -\log_2(P) = -\log_2(1) = 0.$

10.2. A careful consideration of what spontaneous and creative use of language requires has led us to some remarkable conclusions about grammar and the mind. Further work along the lines adumbrated here promises to enrich our view of both ourselves and our world.

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